

2.2 Building Linear Functions from Data; Direct Variation

PREPARING FOR THIS SECTION

- Functions (Section 1.1, pp. 40–51)

Objectives:

- Construct a Linear Model Using Direct Variation

'Are You Prepared?'

$$f(x) = mx + b \quad \text{\S 2.1}$$

$$f(x) = mx \quad \text{or} \quad f(x) = Kx \quad \text{\S 2.2}$$

Lines thru $(0,0)$

$$y = Kx \quad \text{for some constant } K$$

y is directly proportional to x .

y is 7 when x is 5. What's the relationship? What's y when $x=11$?

$$y = Kx$$

K is constant.

$$7 = K \cdot 5$$

$$\frac{7}{5}x \quad \text{No}$$

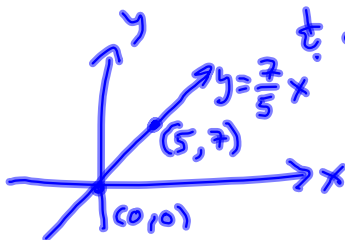
$$(\frac{7}{5})x \quad \text{Yes}$$

$$\frac{7}{5} = K, \text{ so, } y = \frac{7}{5}x$$

$$\frac{7}{5}x \quad \text{Yes}$$

is the relationship

$$\text{when } x=11, y = \left(\frac{7}{5}\right)(11) = \frac{77}{5} = y$$



4 Construct a Linear Model Using Direct Variation

Force is proportional to acceleration.

For an ideal gas held at a constant temperature, pressure and volume are inversely proportional.

The force of attraction between two heavenly bodies is inversely proportional to the square of the distance between them.

Revenue is directly proportional to sales.

Let x and y denote two quantities. Then y **varies directly** with x , or y is **directly proportional to** x , if there is a nonzero number k such that

$$y = kx$$

It's just a line through the origin!

$$p = kB$$

- 20. Mortgage Payments** The monthly payment p on a mortgage varies directly with the amount borrowed B . If the monthly payment on a 15-year mortgage is \$8.99 for every \$1000 borrowed, find a linear function that relates the monthly payment p to the amount borrowed B for a mortgage with the same terms. Then find the monthly payment p when the amount borrowed B is \$175,000.

$$p = kB$$

$$8.99 = k \cdot 1000$$

$$\frac{8.99}{1000} = k \approx .00899$$

$$p(B) = p = .00899 B = \text{payment as a function of } B = \text{Amt borrowed.}$$

$$p(175,000) = (.00899)(175000)$$

$$= \$1573.25 \text{ is monthly pmt.}$$

What's the domain of

$$\frac{25x^{17} - 99x^5}{x^2 - 2x - 15}$$

Need $x^2 - 2x - 15 \neq 0$

$$x^2 - 2x - 15 = 0$$

$$(x-5)(x+3) = 0$$

$$\rightarrow x=5 \text{ OR } x=-3$$

NOT $(x=5 \text{ OR } x=-3)$

Means $x \neq 5$ & $x \neq -3$

$$D = \{x \mid x \neq 5 \text{ \& } x \neq -3\}$$



$$= (-\infty, -3) \cup (-3, 5) \cup (5, \infty)$$

$$= \{x \mid x < -3 \text{ OR } -3 < x < 5 \text{ OR } 5 < x\}$$

Domain of $h(x) = \sqrt{5x-7}$

Need: $5x-7 \geq 0$

$$5x \geq 7$$

$$\left\{ x \mid x \geq \frac{7}{5} \right\} = D = \left[\frac{7}{5}, \infty \right)$$

Domain of $j(x) = \frac{32x-5}{\sqrt{5x-7}}$

Need

$$5x-7 \geq 0$$

Need

$$\sqrt{5x-7} \neq 0$$

combine

$$5x-7 > 0$$

$$D = \left\{ x \mid x > \frac{7}{5} \right\}$$

$$= \left(\frac{7}{5}, \infty \right)$$

$$f(x) + h \neq f(x+h)$$

$$f(x) = x^2 + 13 \rightarrow$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(x+h)^2 + 13 - (x^2 + 13)}{h} \\ &= \frac{x^2 + 2xh + h^2 + 13 - x^2 - 13}{h} = \frac{2xh + h^2}{h} = \cancel{h}(2x+h) = \boxed{2x+h} \end{aligned}$$

$$(x+h)(x+h) = x^2 + xh + hx + h^2 = \underline{x^2 + 2xh + h^2}$$